

NAG Toolbox for MATLAB

s11aa

1 Purpose

s11aa returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$, via the function name.

2 Syntax

```
[result, ifail] = s11aa(x)
```

3 Description

s11aa calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ it is based on the Chebyshev expansion

$$\operatorname{arctanh} x = x \times y(t) = x \sum_{r=0}' a_r T_r(t)$$

where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$, $-1 \leq t \leq 1$, and $t = 4x^2 - 1$.

For $\frac{1}{2} < x^2 < 1$, it uses

$$\operatorname{arctanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right).$$

For $|x| \geq 1$, the function fails as $\operatorname{arctanh} x$ is undefined.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $|x| < 1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The function has been called with an argument greater than or equal to 1.0 in magnitude, for which $\operatorname{arctanh}$ is not defined. The result is returned as zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1-x^2) \operatorname{arctanh} x} \times \delta \right|.$$

That is, the relative error in the argument, x , is amplified by at least a factor $\frac{x}{(1-x^2) \operatorname{arctanh} x}$ in the result.

The equality should hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

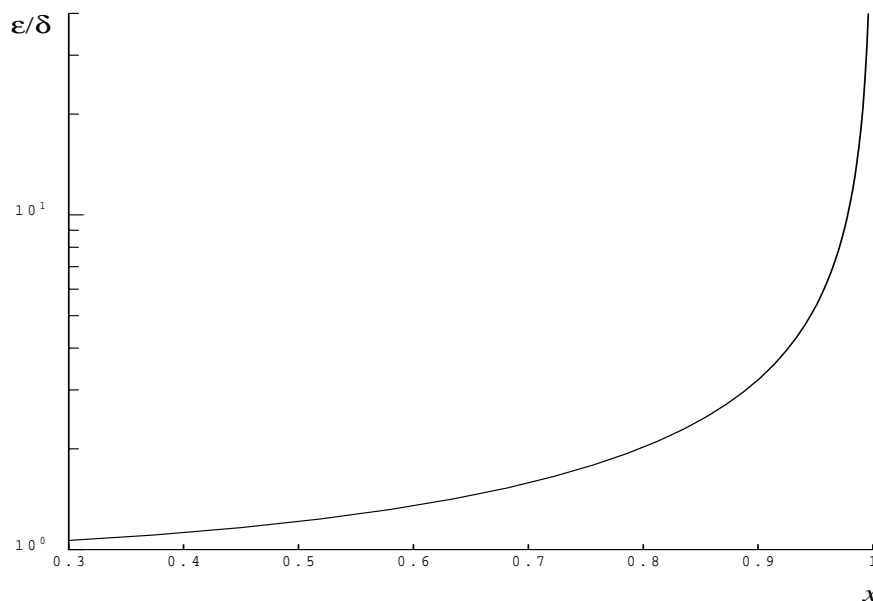


Figure 1

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since x is bounded by definition, $|x| < 1$. In this region where $\operatorname{arctanh} x$ is tending to infinity we have

$$\epsilon \sim 1 / \ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g., if x and 1 agree to 6 significant figures, the result for $\operatorname{arctanh} x$ would be correct to at most about one figure.

8 Further Comments

None.

9 Example

```
x = -0.5;
[result, ifail] = s11aa(x)

result =
    -0.5493
ifail =
         0
```
